# Assessment and Comparison of Various Infiltration Models

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# I. Introduction

The ability to accurately model water infiltration into soils poses immense importance in fields such as watershed management, flood and runoff control, and agriculture. However, this task is not easy to achieve, as the rate of infiltration is comprised of many different components. Soil types, nonuniformity in a soil sample, intensity and duration of precipitation, and hydraulic conductivity ( $K_S$ ) all affect this rate. Under specific assumptions, models have been derived in an attempt to precisely measure infiltration.

In this report, the following models were analyzed and compared: Kostiakov, Horton, Philip, and Green - Ampts. Assumptions were made on the specific soils types, and the experimental data used was provided by Dagadu and Nimbalkar (2012). The soils compared were: sandy, unploughed and ploughed clay, and compacted, harrowed and ploughed black cotton soil (also known as vertisol, and is a heavy, swelling clay that cracks when dry ("IUSS Working Group…", 2015)). Infiltration was measured until the infiltration rate measured became constant. Specific values, as well as a thorough explanation of data collection methods, can be found in the appendix and references.

# II. Problem Statement

As mentioned previously, infiltration rates into soil can be difficult to predict, with various parameters varying greatly between different soil types. The various models chosen for this report each have different errors when comparing soil types as well as each other. Thus the purpose of this report is to see which models best describe the different soil types that were observed in the field data found.

# III. Infiltration Models

## III.I Kostiakov Model

The Kostiakov Model was proposed in 1932, and uses a polynomial function to show the relationship between time and cumulative infiltration. From Brutsaert, if the time being observed is small, then the parameters a and b can sometimes be defined as $a = A_0/2$ , where $A_0$ is the sorptivity of the soil, and $b =- 1/2$ . It was unclear what a small or large time period is, so the constants were determined in matlab from the experimental data. Dagadu and Nimbalkar (2012):

$$F = at^b$$

- $F$ : cumulative infiltration at time t (cm/hr)
- $t$: time (minutes)
- $a$ and $b$ are constants

## III.II Philip's Model

In 1957 Philip solved Richards equation by applying strict conditions and restrictions on the equation, which simplified the equation for infinite infiltration rate and is now known as Philip's two term infiltration model, (Chahinian, et. al, 2005):

$$f(t) = \frac{1}{2}St^{-\frac{1}{2}} + \frac{2}{3}K_s$$

- $f$: infiltration capacity at any time t
- S: Sorptivity of the Soil
- $K_s$: Transmissivity of the soil

The sorptivity of the soil and the transmissivity are either calculated experimentally or as function of moisture capacity or other experimental soil properties. Therefore to fit this model to the data in matlab the equation was simplify to:

$$f(t) = at^{-\frac{1}{2}} + b$$

## III.III Horton's Model

Horton's Model was derived in 1933, and empirically describes infiltration capacity as exponential decay over time from an initial infiltration capacity, $f_0$, and ends at a constant infiltration capacity, $f_c$ (Chahinian, et. al, 2005). $f_c$, in this case, is analogous to the soil hydraulic conductivity. From Dagadu and Nimbalkar (2012):

$$f = f_c + (f_0 - f_c)e^{-kt}$$

- $f$: infiltration capacity at any time t
- $f_c$: final steady state infiltration capacity
- $f_0$: initial infiltration capacity
- $k$: horton's constant representing infiltration rate decay factor in infiltration capacity

## III.IV Green-Ampt Model

The Green-Ampt Model was developed in 1911, and among other things does not account for the capillary force within the moisture profile (Chahinian, et.al, 2005). It is unique to the other models in that it looks at the infiltration capacity as cumulative infiltration, whereas the others look at cumulative infiltration over experient time.

$$f = m + \frac{n}{F}$$

- $f$: infiltration capacity
- $F$ : cumulative infiltration
- $m$ and $n$ are Green-Ampt parameters of infiltration

# IV. Results and Discussion

## IV.I Procedure

The general procedure to find the parameters that fit the infiltration field data was to take the field data from Dagadu and Nimbalkar 2015, and using the necessary transforms (log-log, semi-log), to get a linearized equation for the models and then using matlab's polyfit function to obtain the needed constants. The polyfit function obtains the parameters using a least squares method. Once the constants were determined, they were used to create a best fit line, using the time data or other independent data, that was then compared to the observed field data. The calculated values and the observed values were plotted on the same graph and the root mean square error for each soil and model was calculated. For each soil we decided to have two different scale graphs because the sand and ploughed soils tended to have much larger infiltration rates which did not scale well with the soils that had lower infiltration rates, therefore we plotted the lower infiltration soils with a smaller x and y axis to insure that the data could be presented properly.

As reported in the introduction, the data used in this comparison was taken from field data that was presented by Dagadu and Nimbalkar (2012). However, the parameters that were found in their paper for each model were not reused, and Matlab was utilized to find the constants presented in this analysis. It was unclear how the parameters were determined in the Dagadu and Nimbalkar paper, so a polyfit function was used with the field data to compute the parameters in table 1. Other papers, such as in dos Santos et al. (2016), used an exponential fit with experimental data to find the Kostiakov parameters a and b. It was decided that a polynomial fit would better represent these constants because Kostiakov is a polynomial function. For a detailed description of methods and procedure for reaching these results, see the appendix for full matlab code.

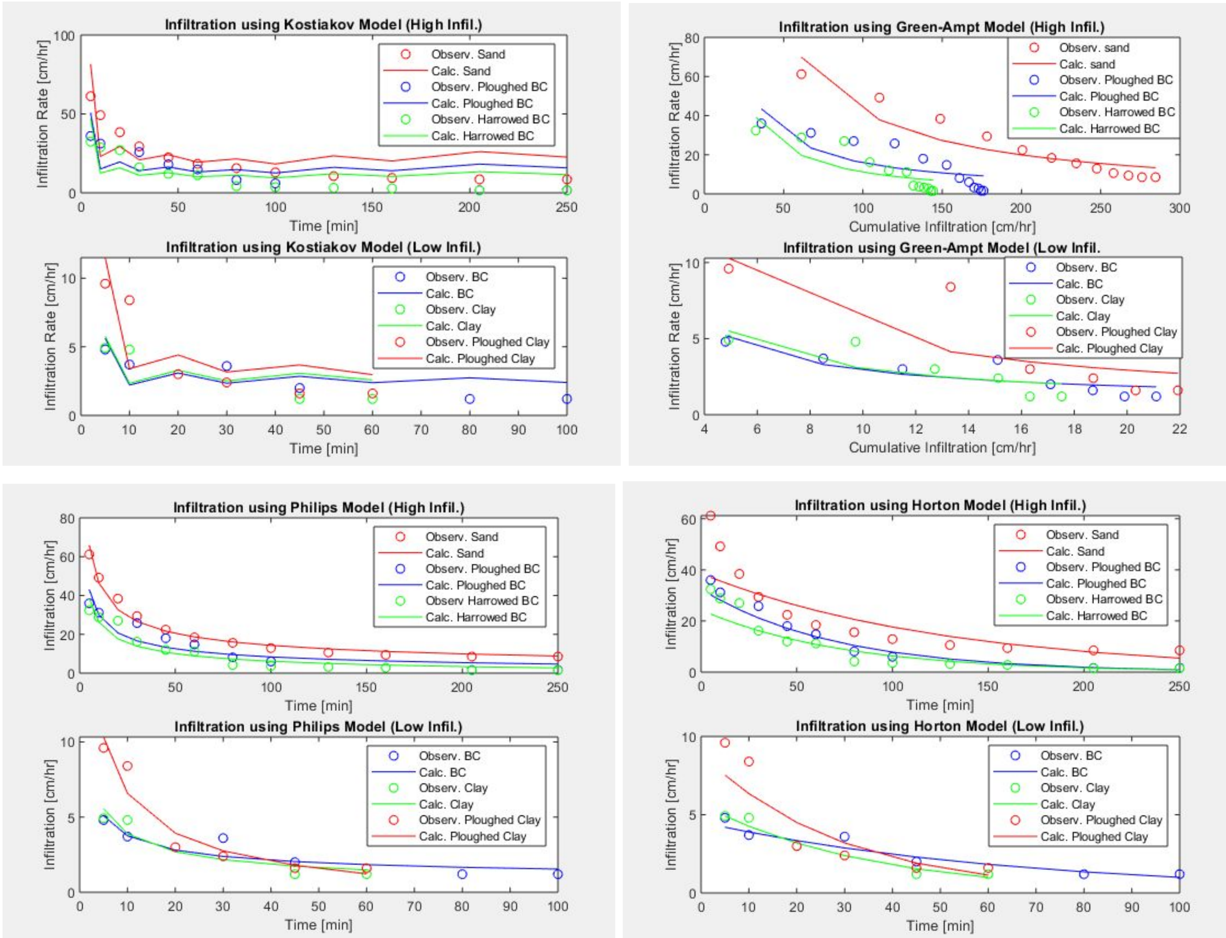
Some of the infiltration models described above required the cumulative infiltration rate, while only the infiltration rate was provided in the field data.
The cumulative infiltration rate was obtained by taking the sum of the individual infiltration rates for each time step. For the Kostiakov model, the data was transformed from infiltration rate, to cumulative infiltration rate to obtain the experimental parameters. Then with the parameters the cumulative infiltration was calculated to compare with the field data, this calculated cumulative infiltration was then transformed back into infiltration rates to compare with the other models, which may

explain why out of all the models our Kostiakov fit was the only fit that was not a smooth function when graphed.

## IV.II Calculated Parameters and Graphs

| **Soil Type** | **Kostiakov** | | **Green-Ampt** | | **Philips** | | **Horton** | |
|---|---|---|---|---|---|---|---|---|
| | **a** | **b** | **m** | **n** | **a** | **b** | **$\ln(i_o - i_c) + \ln(i_c)$** | **k** |
| Sand | 45.8943 | 0.3576 | -2.0975 | 4.4037e+03 | -0.5977 | 148.8847 | 3.6518 | -0.0078 |
| Unploughed Black Cotton | 2.5924 | 0.4799 | 0.8383 | 20.8981 | 0.5175 | 10.2194 | -0.0151 | 1.5108 |
| Unploughed Clay | 2.6109 | 0.4918 | 0.6598 | 23.8235 | -0.1709 | 12.8093 | 1.73910 | -0.029 |
| Ploughed Clay | 6.2971 | 0.3739 | 0.5306 | 47.9987 | -2.4710 | 28.6132 | 2.1913 | -0.0344 |
| Ploughed Black Cotton | 27.8004 | 0.3751 | 0.3862 | 1.5492e+03 | -1.6697 | 100.0979 | 3.4799 | -0.0142 |
| Harrowed Black Cotton | 27.2280 | 0.3393 | -2.2894 | 1.3449e+03 | -3.2270 | 93.4158 | 3.1920 | -0.0135 |

Table 1. Calculated constants and parameters of each model for chosen soil types.




Figures 1 - 4. Results of Kostiakov (1), Green-Ampt (2), Philips (3), and Horton (4). The experimental data was compared with model functions. The sandy soil was put on separate

Note that the Green-Ampt model has different axises to the other three models. As previously mentioned in section II, Green-Ampt uses the cumulative infiltration as the independent variable to calculate the infiltration rate. Visually, it can be observed that Philips and Horton appear to follow similar shapes and match the data well for clay and black cotton types.

Lastly, the root-mean-square error (RMSE) of each model was taken in comparison with the experimental data in order to determine which model was the most accurate for each soil type. Table 2 below lists the calculated RMSE's.

| Soil Type: | RMSE: | | | |
|---|---|---|---|---|
| | **Kostiakov** | **Green-Ampt** | **Philips** | **Horton** |
| **Sand** | 13.2784 | 6.3325 | 2.6124 | 8.7701 |
| **Unploughed Black Cotton** | 1.9991 | 0.6454 | 0.4986 | 0.3939 |
| **Unploughed Clay** | 1.4290 | 0.9087 | 0.5434 | 0.2846 |
| **Ploughed Clay** | 2.4941 | 1.9279 | 0.9140 | 1.3947 |
| **Ploughed Black Cotton** | 11.3116 | 7.3593 | 4.7335 | 2.6682 |
| **Harrowed Black Cotton** | 9.5630 | 6.4866 | 3.7571 | 4.5451 |

Table 2. RMSE of each model for soil types. The lowest RSME for each soil type is labeled in green.

Based on these findings, the most accurate models for sandy, black cotton, and clay soils were: Philips, and Horton.. That being said, Horton had an error of only 0.0085 higher than Philips for compact black cotton soil, and Philips had an additional error of 0.0515 to Horton for unploughed clay soil. Overall, Horton and Philips met the data with closest accuracy with the exception of the sand data. Both of these models incorporate important hydraulic constants in their models. Philip's takes into account the sorptivity and the transmissivity, and Horton scales his model with the initial and final infiltration capacities for the soil in question.

The sand results produced high errors for all the models. An explanation as to why this occurred was not determined. It was not determined as to why this was observed. These models may not be not well suited for soils that have high saturated conductivity.

# V. Conclusion

The goal of this report was to analyze several different infiltration models with different soil types in an attempt to aid infiltration model utilization in industry and other uses. Based on the results, Philips and Horton both performed better and related more accurately with the data.

There are several considerations should the study be developed further in the future. Additional experiments can be taken to collect more data and more soil types could be observed. Other models may be analyzed, such as the Modified Kostiakov, Mezencev, or Richards models. Lastly, initial conditions of the soils could be varied and compared. An example would be changing the initial moisture profile of a soil between wet, normal, dry, wet/normal, etc..

# VI. Contributions of Team Members

Moriah Gilkey contributed to this report with preliminary research, write-up of methods and results, and the entirety of the coding due to her extensive knowledge of Matlab. Jessie Powell contributed to this report with the mini proposal, and write-up of models, applications of infiltration models, and conclusion. The presentation was divided equally.

# VII. References

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# VIII. Appendix

**Table:1. The measured infiltration rates (cm/hr) of different soils under different soil conditions**

| Time (min) | Unploughed clay Soil | Ploughed clay Soil | Black cotton compact Soil | Black cotton ploughed Soil | Black cotton harrowed Soil | Sandy Soil |
|---|---|---|---|---|---|---|
| 5 | 4.92 | 9.6 | 4.8 | 36.00 | 32.40 | 61.20 |
| 10 | 4.80 | 8.4 | 3.7 | 31.20 | 28.80 | 49.20 |
| 20 | 3.00 | 3.0 | 3.0 | 27.00 | 27.00 | 38.40 |
| 30 | 2.40 | 2.4 | 3.6 | 25.80 | 16.20 | 29.40 |
| 45 | 1.20 | 1.6 | 2.0 | 18.00 | 12.00 | 22.40 |
| 60 | 1.20 | 1.6 | 1.6 | 14.80 | 11.20 | 18.40 |
| 80 | - | - | 1.2 | 8.10 | 4.20 | 15.60 |
| 100 | - | - | 1.2 | 6.00 | 3.60 | 12.90 |
| 130 | - | - | - | 3.20 | 3.20 | 10.60 |
| 160 | - | - | - | 2.80 | 2.80 | 9.40 |
| 205 | - | - | - | 1.60 | 1.46 | 8.53 |
| 250 | - | - | - | 1.60 | 1.46 | 8.53 |
| Constant infiltration rate | 1.20 | 1.60 | 1.20 | 1.60 | 1.46 | 8.53 |

Figure 5. Experimental data used in comparison of models. Retrieved from Dagadu & Nimbalkar (2012).

Matlab Code: provided in attached document.